

Math 116 Section 04

Quiz 11

Name _____

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Student Number _____

All solutions are to be presented on the paper in the space provided.
The quiz is open book. You can discuss the problem with others and
ask the TA questions.

Evaluate the following integrals:

(1)

$$\begin{aligned}\int_{-\infty}^0 e^x dx &= \lim_{t \rightarrow -\infty} \int_t^0 e^x \\ &= \lim_{t \rightarrow -\infty} e^x \Big|_t^0 \\ &= \lim_{t \rightarrow -\infty} (e^0 - e^t) \\ &= 1 - 0 \\ &= 1\end{aligned}$$

(2)

$$\begin{aligned}\int_1^\infty \frac{1}{x} dx &= \lim_{t \rightarrow \infty} \int_1^t \frac{1}{x} dx \\ &= \lim_{t \rightarrow \infty} \ln|x| \Big|_1^t \\ &= \lim_{t \rightarrow \infty} \ln t - \ln 1\end{aligned}$$

The first term doesn't exist, so the integral doesn't converge.

(3)

$$\begin{aligned}\int_{-4}^0 \sqrt{-x} dx &= \lim_{t \rightarrow 0^-} \int_{-4}^t \sqrt{-x} dx \\&= \lim_{t \rightarrow 0^-} -\frac{2}{3}(-x)^{\frac{3}{2}} \Big|_{-4}^t \\&= \lim_{t \rightarrow 0^-} -\frac{2}{3}((-t)^{\frac{3}{2}} - (--4)^{\frac{3}{2}}) \\&= -\frac{2}{3}(0 - 8) \\&= \frac{16}{3}\end{aligned}$$

(4)

$$\begin{aligned}\int_{-1}^1 \frac{1}{x} dx &= \int_{-1}^0 \frac{1}{x} dx + \int_0^1 \frac{1}{x} dx \\&= \lim_{t \rightarrow 0^-} \int_{-1}^t \frac{1}{x} dx + \lim_{t \rightarrow 0^+} \int_t^1 \frac{1}{x} dx \\&= \lim_{t \rightarrow 0^-} \ln|x||_t^{0^+} + \lim_{t \rightarrow 0^+} \ln|x||_0^t\end{aligned}$$

Since $\lim_{t \rightarrow 0^+} \ln|x| = -\infty$, the limit does not exist and the integral diverges.